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A Method to Predict Atomization Performance in Gas-Centered Swirl-Coaxial Injectors

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Abstract

The ability to predict atomizer performance can reduce the cost of system development in many areas. This paper investigates the atomization efficiency and droplet distribution from films with strong gas-phase influences. A prediction of atomization efficiency based on a general theory of the droplet creation process is given. In this process a disturbance is created on the film surface then broken down into droplets via stripping. The theory relates the mass of film lost via atomization to the mass of liquid introduced into the atomizer to predict atomization efficiency and offers some estimations of primary droplet diameter. A specific example involving a gas-centered swirl coaxial injector is given to illustrate how the theory would be applied; however, efforts are made to keep the model as general as possible so that it applies to many types of atomizers and a wide range of operating conditions.

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Introduction

The introduction of liquid and its subsequent atomization are important in a wide variety of process and subprocesses. Understanding atomization is therefore important for predicting the performance of many systems. Atomization is generally viewed as a device-dependent operation, i.e. atomization quality and character is considered to be dependent on the specific type of atomizer utilized. A recent review of the mechanisms effecting film atomization [1] brings to light a device-independent view, however. This more general viewpoint has been applied to develop an atomization model based on general atomization mechanisms. While this theory does not apply to all atomizers, it may be utilized to predict the performance of many different systems provided they fall within certain classes. Specifically, the theory developed here-in applies when droplets are produced from a wall-bounded film and gas-phase effects are important due to high-speed gas flows, high pressures or both. As will be shown, the importance of the gas phase can be estimated via a momentum-flux ratio.

The specific injector motivating this study is a gas-centered swirl-coaxial (GCSC) injector. A schematic of the injector is shown in Fig. 1. Liquid is injected tangentially along the outer wall of the atomizer. This tangential injection causes a swirling film to form along the wall. High-speed, nonswirling gas is introduced axially through the center of the injector. This injector is an effective atomizer because the gas flows at a much higher speed than the liquid, and, therefore, the gas momentum flux is larger than the liquid momentum flux. These injectors share characteristics with other atomizers such as pressure-swirl and coaxial air-blast atomizers, but unlike these other injectors they do not produce a conical sheet at typical operating conditions. Instead, atomization occurs from a wall-bounded film inside the injector cup [2, 3]. More information on this injector as well as some experimental and numerical simulation results may be found in an earlier ILASS paper [2]. Following the general theoretical development an example is given using this atomizer.

As mentioned above, this work is predicated on an earlier examination of atomization mechanisms in films where several likely atomization mechanisms were determined [1]. Numerical and experimental testing of a GCSC injector has further reinforced the identifications made in this earlier work [2]. In the current model droplets are produced as a result of the stripping of various disturbances. These disturbances may arise from different initiating mechanisms such as liquid turbulence or hydrodynamic instabilities. This paper describes the stripping process and develops a theory for calculating the rate of atomization, atomization efficiency and primary droplet size. As in prior simplified models describing film breakup in cooling tubes, this theory assumes that liquid is stripped from a disturbance when the forces on a section of the disturbance are balanced [4, 5].

The theoretical development is presented in two subsections. The specifics of stripping and how to quantify it are given first. These details form the main backbone of the theory which is intended to be as general as possible: it applies to any reasonably sized disturbance in a flow which is, on the average, parallel to the bulk liquid interface. The second subsection presents the procedure for calculating atomization efficiency. Assumptions and procedures for calculating atomization rates and primary droplet sizes are also given. Following the presentation of the general theory, a specific example is given for a GCSC injector. This example serves as a medium for discussing various aspects related to the practical application of the theory.

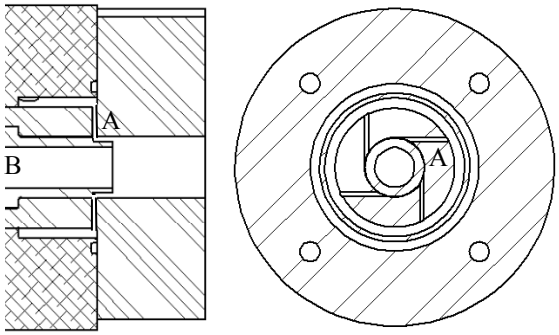

Figure 1. Partial schematic of the experimental apparatus. The liquid enters through the tubes labeled A; gas enters at B. The gas inlet extends beyond what is pictured.

Theoretical Development

Stripping

In the simplest sense stripping can be considered to occur as one of three different modes—"pull", "push" and "scoop" (Fig. 2). In "pull" stripping the lift over a curved surface causes it to further distort and eventually lifts a section of liquid from the film [6]. "Push" stripping is the result of drag forces where a section of a disturbance is dragged from the film [5]. "Scooping" is the result of gas-phase structures, such as a recirculation zone, where the gas motion causes a distortion in the liquid—distorting it to the point that part of the liquid is separated from the bulk [2]. "Scooping" can be thought of as the result of friction instead of lift or drag.

Figure 2. The three modes of stripping are illustrated here: (a) "push", (b) "pull", (c) "scoop".

In realistic situations, however, stripping is an amalgamation of the three modes since all liquid protuberances are subject to lift, drag and other forces. Despite these forces acting on a disturbance, it might not undergo stripping. The criterion determining if stripping occurs is based on a simple assumption: if the forces over a section of the disturbance are balanced then that section will separate from the bulk film. This criterion has been successfully used by several other researchers to calculate atomization rates (due to "push" stripping only) in cooling tubes with multiphase annular coflows [4, 5].

Figure 3 shows a set of forces on a generic disturbance. The general direction of the forces is indicated where the angles are only approximate. As shown in the figure lift, drag and friction forces aid atomization while surface tension, centripetal and viscous forces impede the separation. Separation occurs at some angle to the radial direction, θ_s. The balance of forces should, therefore, be considered not in the radial or axial directions but along the angle θ_s.

$$\begin{aligned} &Lift + Drag\left(\tan\theta_s\right) + Friction\left(\cos\theta_f + \sin\theta_f \tan\theta_s\right) \\ &- Centripetal - Surface\,Tension\left(\cos\theta_\sigma + \sin\theta_\sigma \tan\theta_s\right) \\ &- Viscous\left(\cos\theta_\mu + \sin\theta_\mu \tan\theta_s\right) = 0 \end{aligned} \qquad (1)$$

Since the trigonometric terms are well-behaved over a wide range of angles (until the angles approach 90°), the exact angle need not be specified: each trigonometric term may be considered a constant of O(0.1) to O(1). Note that while this formulation is rather general including forces which may not occur in all situations (e.g., friction or centripetal forces), it is slightly simplified in that gravity has been neglected. The gravity term may be easily incorporated, but care must be taken that the term is included in the correct direction as it may either contribute to or hinder atomization.

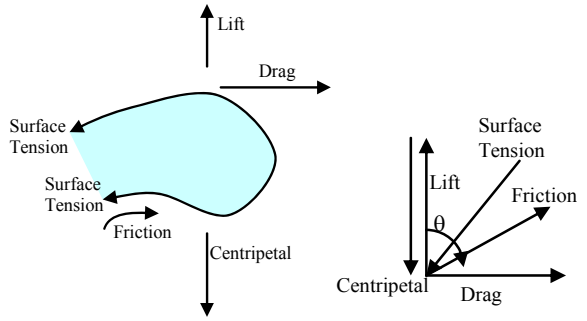

Figure 3: A depiction of the forces acting on the separating portion of a disturbance

The expressions for the forces need not be exact; because trigonometric constants are already present, adding proportionality constants for each force does not appreciably increase the formulation's complexity. As a result, lift and drag may be combined into a single term since both forces are proportional to

$$\rho_g A_{cs} v_{rel}^2 \qquad (2)$$

The friction term is similar, but the specific area and velocity involved differ

$$\rho_g A_{surface} v_{rel,recirc}^2 \qquad (3)$$

Note that the surface area is not that of the entire film or disturbance, but the area in contact with the recirculation zone or other flow feature causing the friction force. Similarly, the surface area in the viscous force term is for the area in contact with the distorting flow field.

$$\mu_\ell A_{surface} v_{rel} / \delta \qquad (4)$$

The surface-tension force is proportional to

$$\sigma A_{\perp atom} / R_{curv} \qquad (5)$$

The area perpendicular to atomization is an approximation and is rather difficult to determine since the atomization angle is not resolved. Consequently, this area will be simplified to either the annular area occupied by the disturbance or area of the disturbance seen by looking along the axis of the gas flow. Finally, the centripetal acceleration term is proportional to

$$\rho_\ell V_{ent} a_{centripetal} \qquad (6)$$

with the centripetal acceleration given by $a_{centripetal} = v_{local,\theta}^2 / r_{rot}$. If included, the gravity term has the same form as the centripetal acceleration term with $a_{centripetal}$ replaced by the gravitational acceleration. The various areas, entrained volume and geometric parameters depend on the disturbance shape and atomizer geometry (e.g., annular or flat).

Utilizing the above expressions the force-balance equation contains fluid properties, gas and liquid velocities, and geometric parameters of the disturbance. Ideally, all of these are known except one geometric parameter—the atomized section height, δ—for which the equation is solved. Practically, however, the exact velocities and disturbance shapes are rarely known. Simulations or experiments could be used to develop these values over a range of conditions, but this approach is unlikely to be exhaustive and would be time and cost intensive. It is recommended instead that assumptions be made in order to approximate these not-fully-known values. In particular, the shape of each disturbance can be presumed in order to calculate the various geometric parameters found in the force terms; for example, a disturbance caused by hydrodynamic instabilities could be assumed to have a sinusoidal cross section. The relative velocity can also be approximated as the gas velocity if this is much greater than the liquid velocity. Introducing this approximation for the relative velocity and nondimensionalizing with $\rho_1 v_{in}^2 \tau_{gap}^2$, the force-balance equation can be written as

$$\Phi\left[C_{L/D}\,A_{cs}/\tau_{gap}^2 + C_f\left(v_{g,recirc}/v_g\right)^2\left(A_{surface}/\tau_{gap}^2\right)\right]$$
$$- C_c\,V_{en}v_{local,\theta}^2/\left(r_{rot}\tau_{gap}^2 v_{in}^2\right) - C_\sigma We_\ell\,A_{\perp atom}/\left(R_{curv}\tau_{gap}\right)$$
$$- C_\mu\,A_{surface}v_g/\left(Re_\ell v_{in}\tau_{gap}\delta\right) = 0 \qquad (7)$$

where Φ is the momentum-flux ratio of the gas and liquid at inlet conditions, $We_\ell = \sigma/\left(\rho_\ell \tau_{gap} v_{in}^2\right)$ is the liquid Weber number and the Reynolds number of the liquid is $Re_\ell = \rho_\ell \tau_{gap} v_{in}/\mu_\ell$. This equation highlights the importance of the momentum-flux ratio, as all terms aiding stripping are multiplied by it. Consequently, the assumption that the gas velocity is much greater than the liquid velocity seems justified until the liquid and gas density approach one another.

General Procedure to Calculate Atomization Rate

The first step in determining the atomization efficiency and/or average droplet diameter is to develop the necessary experimental measurements, simulations or assumptions to determine what type or types of disturbances are present on the film. The type of disturbance depends on the operating conditions and geometry of the atomizer and is related to creation mechanisms as outlined in an earlier work [1]. For example, liquid turbulence creates one type of disturbance (ligaments) while hydrodynamic instabilities create another (waves). In some instances a single type of disturbance will be present; in other cases numerous types will exist on the same film. A set of sample questions for determining the type of disturbances present is presented as Table 1.

Once disturbance types are determined, the velocities and geometric parameters appearing in the force-balance equation must be established for each disturbance type present. Again, these values may be determined through measurements, simulations, assumptions or a combination thereof. Generally, common sense and prior experiments and simulations with similar atomizers or conditions will allow good choices of disturbance shapes. Once a shape is chosen, the geometric parameter can be determined through application of geometry and simple calculus. The determination of the radii of curvature and rotation need a few additional notes, however. Two main choices present themselves for approximating the radius of rotation: set this radius equal to the average radius over the atomized section of the disturbance (simple) or set it equal to the radial value of the cross-section's centroid (accurate). The radius of curvature is more complex and an obvious assumption does not present itself. For ease and because of its successful use in earlier force-balance formulations [5], the value at the crest of the disturbance is recommended as an approximation for the value at atomization.

Can aerodynamic effects clearly be neglected? $\Phi \le O(1)$*
YES—this mechanism does not apply
NO—move on to the disturbance type questions
Stripping from liquid turbulence disturbances possible? (must answer yes to all below)
Is the liquid turbulent?
Do the eddies have sufficient energy to deform the surface ($We_{eddy}>O(1)$)?
Is the outlet length sufficient for the eddy to form?
Stripping from hydrodynamic instabilities waves possible? (must answer yes to all)
Is the flow unstable or transiently unstable?
Is the injector of sufficient length for a wave to grow to a height where it could be atomized?
Are disturbances due to a recirculation zone possible? (must answer yes to all)
Is there a potential recirculation zone due to the geometry of the injector (step, steep slope, other area of possible separation)?
Is the recirculating flow energetic enough? ($\Phi(v_{re}/v_g)^2>O(1)$*)
Are disturbances due to gas-phase turbulence possible? (must answer yes to all)
Is the gas turbulent?
Is the flow energetic enough to be important? ($\Phi\,(v_{rms}/v_g)^2>O(1)$*)

*in some cases the criterion will be different

Table 1: Sample questions for determining possible disturbance types. Note that existence of a disturbance does not guarantee stripping of the disturbance. For example, in strong gas flows ligaments produces by liquid turbulence may topple before they can be stripped.

Velocity determinations tend to be atomizer specific and require either a large body of experimental/simulation work or many assumptions. However, progress can be made with straightforward approximations. For example, the local tangential velocity of the liquid may be considered to vary linearly with distance. The gas velocity may be equated with that of the incoming gas, related to the distance via correlations for flow over wavy surfaces [5] or estimated from single-phase simulations of the atomizer [7]. Experimentally measured values could also be used, but, unless some general correlation can be determined from a limited number of experiments, this approach would be very time consuming and costly.

Once the velocity and geometric values are established, Eq. 7 may be solved for the height of the disturbance lost to atomization, δ. This lost height along with the shape approximation gives a value for the entrained volume. The calculated entrained volume is that lost by an individual disturbance. Several protrusions may be present on the film, however, so the number of each type of disturbance must also be ascertained. This

number is related to the film length, volume or surface area of the liquid and the disturbance type. Axial film velocities and atomizer geometry may also play a role.

The total atomized volume is the sum of the liquid atomized from all of the disturbances. The simplest approach is to estimate the stripping as a global, i.e. time-independent, value. In most cases, however, the stripping is cyclic since a disturbance forms, grows, loses mass, grows and loses mass again. An unsteady formulation, while more complex, allows this shedding frequency to be estimated. A time-dependent approach often makes the calculation of the number of disturbances on the film more straight-forward, particularly in the situation where disturbances grow as they travel downstream. Waves created by hydrodynamic instabilities are one example of such a disturbance. The growth-rate of a disturbance varies depending on the cause of the disturbance. The time necessary for a section of liquid to be stripped from the disturbance (once the disturbance reaches a particular height) is assumed to be the time for the gas to flow over the atomizing portion of the disturbance. Using the knowledge of the number of disturbances on the film, their growth rate and an atomization time allows the calculation of a volumetric atomization rate—

$$Q_{atom} = \sum_{disturb\_type} N_{disturb} V_{en} / t_{atom} \qquad (8)$$

The atomization efficiency is found by comparing the volumetric flow rate of the incoming liquid with the atomization rate from all disturbances. In most cases, a film length is needed to arrive at the total number of disturbances. To get the correct atomization rate, the correct film length must be used. This film length is unknown at the start of the calculations and must be estimated. If the atomization rate calculated from the estimated film length is not equal to the inlet flow rate then the film length is incorrect and a revised estimate for the film length should be used to get a new atomization rate. This iteration process continues until the two flow rates are equal. Note, however, that the film length cannot exceed the atomizer length as this theory is built on the assertion that atomization occurs from a film only. Once the correct film length has been determined then the atomization efficiency can be calculated as [7]

$$\eta_{atom} = \left(L_{inj} - L_{film}\right) / L_{inj} \qquad (9)$$

Atomization efficiency is only one part of atomizer performance; the droplet size is another important parameter. The force balance results in the calculation of an atomized volume from each disturbance. Stripping is assumed to occur across the entire disturbance which may be the length of the film or some shorter distance. Many types of disturbances will, therefore, produce a ligament when stripping occurs; the length of these ligaments equals the disturbance length. For example, an annular disturbance would produce a toroid-shaped ligament. Using the ligament length and the calculated entrained volume, the diameter of the ligament is

$$d_{lig} = \sqrt{V_{en} / \left(\pi L_{lig}\right)} \qquad (10)$$

(Note that for if a torus is produced L_{lig} is the circumference at the center-point of the atomized section.) Assume that this ligament breaks up via the Rayleigh mechanism, so that it yields droplets with a diameter ~1.89 times the diameter of the ligament [8]. The number of droplets produced by this ligament is roughly

$$N_{drop} = 6V_{en} / \left[\pi \left(1.89 d_{lig}\right)^3\right] \qquad (11)$$

This droplet-number equation may be used in all cases, even when a single droplet is produced; in that case the number of droplets calculated will be one or less. When the number of droplets is greater than one but not an integer, the ligament should be assumed to break into somewhat differently sized droplets in order to generate an integer number. In other words, the number of droplets should be rounded to the nearest integer. The droplet diameter is then

$$d_{drop} = \left[6V_{en} / \left(\pi N_{drop}\right)\right]^{1/3} \qquad (12)$$

where N_{drop} is the integer value. As with the entrained volume calculation, the entire droplet size procedure is applied to each type of disturbance. The average droplet diameter produced by primary atomization is a weighted average of the calculated droplet sizes

$$d_{avg} = \sum_{disturb\_type} N_{drop} d_{drop} \Big/ \sum_{disturb\_type} N_{drop} \qquad (13)$$

where the summations are taken over all disturbance types. A summary of the entire process is given in Table 2.

Results and Discussion

As pointed out above, the application of this theory requires information that is not readily known *a priori* and may be specific to the atomizer. In order to demonstrate this theory in action, therefore, an example is given of a specific atomizer—a GCSC injector. The geometry and general operation of a GCSC injector are described in the introduction and shown in Fig. 1. Due to space constraints and its illustrative nature, this example case is simplified by considering only a small range of operating conditions where a single type of disturbance, one caused by gas-phase turbulence, is important. In this regime eddies in the gas are so energetic that they distort the surface, producing protrusions. Example operating conditions in this range as well as some nondimensional parameters are given in Table 3.

The first information that depends on the specific atomizer (and is often not fully known) are the relative and liquid-tangential velocities. The gas velocity, which is purely axial in this example, is much greater

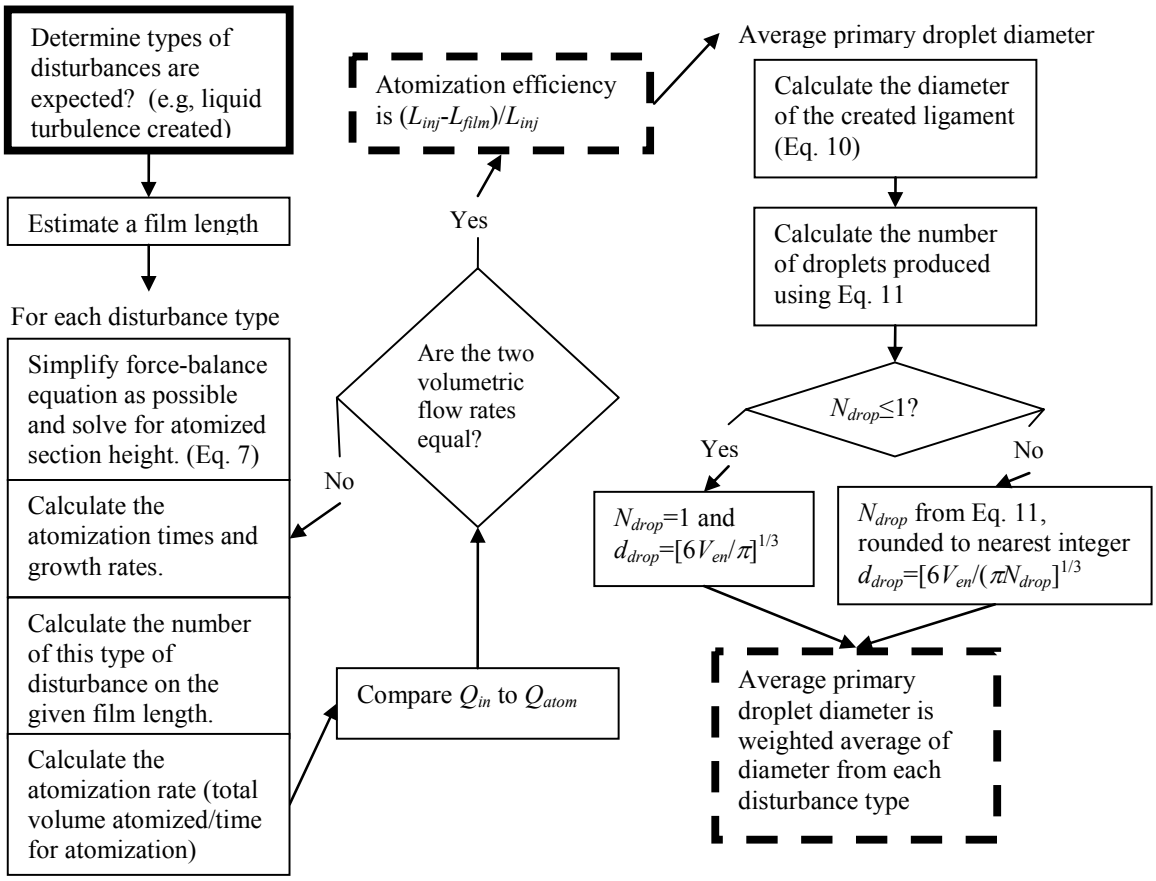

Table 2: A summary of the basic process for determining atomization efficiency and average droplet diameter.

than the liquid velocity. Therefore, assume the bulk relative velocity is also purely axial and equal to the gas velocity. Earlier work on a similar atomizer suggested that the appropriate gas velocity is not its bulk velocity but its value near the interface which can be approximated from a single-phase numerical simulation of the injector [7]. The relative velocity of recirculating gas, important in the friction term, arises from turbulent eddies and can be estimated from turbulent statistics such as the root-mean-square velocity fluctuation in the axial direction. Multiphase, VOF simulations (see Fig. 4) suggest that the liquid-tangential velocity falls sharply following liquid injection and that the ratio of tangential to inlet velocity is O(1); an initial approximation might assume $v_{local,\theta}=C_{\theta}v_{in}$. From simulations a more accurate functional dependence between the two velocities could be developed; however, these simulations are quite time consuming and given the other assumptions, unlikely to appreciably improve accuracy.

Assume that the eddies contacting the liquid surface are spherical or cylindrical in shape. Turbulence literature [9] suggests the shape may be quite a bit more complex; yet, atomization literature studying atomization due to liquid turbulence has been quite successful

Gap Thickness (in)	0.065
Outlet Radius (in)	0.375
Liquid Mass Flow (lb/s)	0.101
Gas Mass Flow (lb/s)	0.135
Liquid inlet velocity (ft/s)	10-30
Gas inlet velocity (ft/s)	1000 +
Φ	3 - 5
We_l	$O(10^{-3})$
Re_l	$O(10^{4})$

Table 3: Operating conditions for the GCSC injector. Working fluids are gaseous nitrogen and water; test conducted at STP.

using simplified eddy shape and behavior [8]. The area and volume calculations are most straightforward if the eddy is assumed to be cylindrical; a rough length estimate may be obtained from tangential velocity fluctuations or by assuming the eddy is somewhat equivalent to a sphere and, therefore, has a length equal to half its cross-sectional circumference. This rotating eddy pulls liquid up along its outer edge in a manner similar to a rotating dip coater. The resulting disturbance is illus-

trated in Fig. 2c; as seen in the figure, determining the relevant areas involves several unknown parameters such as R_{eddy}, a distance from the center of the eddy to the nominal film surface (y) and a radius of the outer portion of the disturbance (R_{out}). The eddy radius can be determined from turbulence statistics and the other parameters can be calculated by applying simple, logical conditions. For example, the volume of fluid displaced above the original surface is equal to the volume of the divot (the void below the undisturbed film surface). Secondly, assume that the liquid is drawn to the top of the eddy at the time of atomization then $h=R_{eddy}+y$. (This conditions means the center of the outer circle must lie on the same vertical line as the circle describing the eddy.) To fully describe the geometry one final condition is necessary or the force-balance equation may be solved to get δ as a function of one of the geometric parameters, such as h or R_{out}. Calculations using simplified geometries, the values cited in Table 3, v_{rms} equal to 30% of the bulk gas velocity, several values of R_{eddy} starting from the minimum discussed in the following paragraphs and a range of h indicate that h must be relatively large, larger than R_{out}, before atomization occurs. If a definitive answer is desired rather than a possible range as h varies, a condition equating the kinetic energy of the eddy and the surface energy of the overall disturbed shape is suggested.

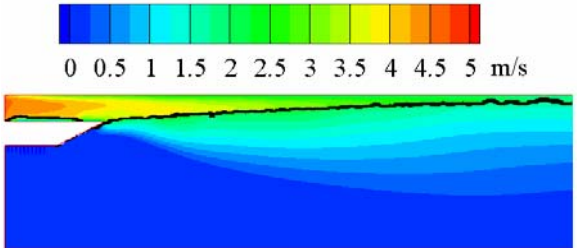

Figure 4: The tangential velocity plot from an axisymmetric, two-phase simulation of the injector. The black line represents the approximate location of the interface. Note that this figure is for a gas velocity around 100 m/s only.

The force-balance equation can now be formulated in terms of two still-unknown parameters—δ and R_{eddy}. In many cases the problem may be simplified by neglecting certain forces that are much smaller than the others. Forces can be calculated with δ values near 0 and near h using the operating conditions and either the above complex geometry or a simplified but similar one (e.g., a triangle of height δ and a width similar to the atomized portion of the disturbance). If any force is small over the range of δ it may be neglected; other forces may be neglected if small at either limit and then rechecked after a value of δ is calculated. If all of the positive terms are negligible then stripping does not occur. Somewhat surprisingly, the sample calculations mentioned above (parameters as in Table 3, $v_{rms}=0.30v_g$ and a range of h and R_{eddy}) indicate that the surface forces are negligible in most cases, generally being two or more orders of magnitude smaller than viscous and centripetal forces.

The amount of liquid atomized by a single disturbance may now be calculated if the general size of the eddy is known. Since this calculation is for a single eddy, the number of disturbances must also be determined in order to calculate the total atomized volume. Clearly, the size and number of eddies depends on the flow conditions and type of turbulence. In this case turbulence is a result of developed pipe flow. In order to distort the interface the energy of the turbulent eddy must exceed the surface energy of the film, $d_{eddy} \sim \sigma/\rho v_{eddy}^2$; this follows the theoretical development predicting disturbance sizes due to liquid-phase turbulence [8]. If the eddies responsible for disturbance formation lie in the inertial range, as they do for liquid turbulence, then $v_{eddy} \sim u_{rms}(d_{eddy}/d_I)^{1/3}$ where the root-mean-square velocity is in the cross-stream direction and d_I is the integral length scale, $d_H/8$ [8]. Eddies responsible for atomization therefore meet the following criterion $d_{eddy}/d_H = C_{eddy}(\sigma/\rho d_H v_{eddy}^2)^{3/5} = C_{eddy} We_{dH,eddy}^{3/5}$, at a minimum, with the proportionality constant O(1). Starting with this minimum eddy size, with a constant of unity, results in a primary droplet size around 20-150 μm. Droplet size measurements with similar atomizers [7] found SMDs less than 75 μm to as fine as 5 μm. Given that these measurements were made downstream of the atomizer exit, after secondary breakup occurred, the theory's findings seem reasonable. The percentage of eddies meeting the criteria for producing distortions as well as the average distortion-producing eddy size can be estimated from the turbulence spectrum of the atomizer or a similar configuration, such as a rough pipe.

Of the eddies capable of deforming the surface only a fraction actually impact the film; this fraction is related to the amount and profile of liquid in the injector. Imagine that the profile of the film is roughly linear with a maximum at the lip edge where the film thickness is the gap thickness, $\tau_{local}=\tau_{gap}(1-z/L_{film})$. The film volume is

$$V_{film} = \pi L_{film} \tau_{gap} \left(3r_0 - \tau_{gap}\right)/3 \qquad (17)$$

Assume that the ratio of the film volume to total volume is equivalent to the fraction of existing eddies to eddies which impact the film. The total number of disturbances formed is

$$N_{disturb} = \frac{1}{3}\frac{L_{film}}{L_{inj}}\frac{\tau_{gap}\left(3r_o - \tau_{gap}\right)}{r_o^2}\left(\%_{capable} N_{eddy}\right) \qquad (18)$$

Unfortunately, the overall film length is not initially known and must be guessed in order to calculate these parameters. Simulations, experiments or prior calcula-

tions are suggestions for developing an initial guess of the film length. In the absence of other information the injector length is a good starting point.

As for atomization times, suppose that at any instant the total number of impacting eddies, $N_{disturb}$, are in contact with the film. These eddies require a finite time to distort the film and achieve atomization; the next batch of eddies then contact the film after some lag. In actuality, this type of disturbance formation and atomization are continual processes with no set atomization frequency; however, this simplification makes the calculation of an atomization time more straightforward. Forming the distortion requires a time of $\theta R_{eddy}/v_{g,recirc}$—the time for the circulating flow within the eddy to rotate up along the disturbance height. The atomization event time is that needed for the circulating flow to move across the atomizing portion of the disturbance. The lag time between batches of eddies is needed to account for the reality that all of the eddies do not contact the film at once. It may be approximated as an average traveling time for an eddy to move from its position in the gas to the liquid surface, $(r_o/2)/u_{rms}$. Overall, then, the process is approximated as $N_{disturb}$ atomizations occurring every $\theta R_{eddy}/v_{g,recirc}+\delta/v_{g,recirc}+r_o/(2u_{rms})$ seconds. Combining the above information

$$Q_{atom}=N_{disturb}V_{en}/t \qquad (19)$$

where $t=\theta R_{eddy}/v_{g,recirc}+\delta/v_{g,recirc}+r_o/(2u_{rms})$.

The calculated atomization rate is compared with the volumetric flow rate of the liquid into the injector. If these values are not equal then the film length is incorrect and a new estimate is needed. This estimate may be based on the atomization rate directly, or it may be calculated by considering the difference in volumetric flow rates. The first choice is slightly less complex, but tends to overestimate the difference between the lengths and require more iterations. To calculate a film length based on the difference in volumetric flow rates, let $\Delta Qt_{flow}=\Delta V$. The flow time can be represented by $L_{film}/\overline{v}_{local,z}$ where the bar represents an average value over the film's length. The average axial velocity is the integral of the local axial velocity over the film length. Setting the volumetric flow rate through the r-θ plane as a constant equal to the inlet flow rate and further assuming the atomization is evenly distributed over the film gives an expression for the local velocity:

$$v_{local,z}=Q_{in}/A_{local} \qquad (20)$$

A_{local} is just the annular area from the pipe wall to τ_{local}. The volume difference is Eq. 17 with L_{film} replaced by ΔL_{film}. The difference in flow rate, ΔQ, is just Q_{in}-Q_{atom}. Combining all of these pieces allows an equation for ΔL_{film} to be developed.

Once the calculated atomization and inlet flow rates are equal then the film length has been correctly calculated. The atomization efficiency is then given by Eq. 9. Since only one disturbance type is considered the average droplet diameter due to primary atomization may be found by solving the force-balance equation for d and using the entrained volume formula along with a ligament length of $\pi d_{eddy}/2$.

Following the above outline, if the turbulence parameters are known or may be determined (say from prior experiments/DNS of fully developed pipe flow) then the atomization efficiency and average primary droplet size may be determined. As indicated above, sample calculations suggest primary droplet sized of 20-150 μm for the atomizer examined here. The size and amount of disturbance atomized suggests that up to 40% of the liquid's surface must be covered with atomizing disturbances to fully atomize the film over the length of the atomizer. Since the ratio of film to total volume is only 25% this suggests that the primary droplet size is likely to be in the upper value of the calculated range and that eddies larger than the minimum necessary size likely play an important role in atomization.

Conclusions

A theory has been developed which uses an understanding of general atomization mechanisms to calculate atomization properties of devices utilizing films and having strong gas-phase involvement. A general outline of the process was given as well as an example from a specific atomizer operating in a restricted regime. The theory assumes that atomization occurs through a process of surface disturbance creation followed by the gas-phase-initiated stripping of a portion of these disturbances. The model is applicable to any number of disturbance creation mechanisms; only one, that due to gas-phase turbulence, was considered in detail here. Stripping occurs when the forces acting on a small section of a disturbance are balanced. This small section separates from the bulk fluid, forms a ligament and, eventually, droplets. Application of the theory requires several assumptions, specifically determining velocities and disturbance shapes. Disturbance shapes can often be estimated rather accurately while the velocity estimations remain a large source of uncertainty. The uncertainty may be mitigated through experimental studies or by the use of a limited number of simulations to determine suitable velocity relations. Overall, however, the theory is powerful in its ability to offer predictions for a wide range of atomizers operating over a wide range of conditions. Initial calculations are promising, predicting primary droplet diameters in an expected range. The model also fills a void in that it deals with the film, or wall-bounded liquid, configuration which has received relatively little attention and, therefore, has few empirical correlations or analytical prediction methods.

A large amount of future work is planned. Much work is needed to validate this model and find suitable ranges for the constants contained within it. Work is also planned to further elucidate the effects of various assumptions and to allow recommendations of suitable assumptions over a wide range of conditions and geometries. The theory's sensitivity to various operating parameter will also be explored.

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Nomenclature

a	acceleration
A	Area
C	constant
d	diameter
h	overall height of a disturbance
L	length
N	number of things
Q	volumetric flow rate
r,R	radius
Re	Reynolds number
t	time
u	radial velocity
v	velocity
V	volume
y	distance, see Fig. 5
z	axial coordinate
We	Weber number
δ	height of the atomized portion of the disturbance
η	efficiency
θ	an angle
μ	viscosity
ρ	density
σ	surface tension
τ	thickness
Φ	momentum-flux ratio

Subscripts

$\perp$ atom	perpendicular to atomization
atom	atomization
avg	average
cs	cross-section
curv	curvature
disturb	disturbance
drop	droplet
eddy	turbulent eddy
en	entrained
f	friction
g	gas
gap	of the gap prior to where the liquid and gas meet
in	inlet value of liquid
inj	injector
l	liquid
L/D	Lift and Drag
lig	ligament
recirc	recirculation
rel	relative
rot	rotation
s	at which stripping occurs
o	outlet of the injector
z	in the axial direction
recirc	recirculation
rel	relative
μ	viscous
θ	in the angular direction
σ	surface tension

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